Peripheral Contributions to the Difference of Particle and Antiparticle High-Energy Cross Sections*

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One-pion-exchange contributions to the differences between total cross sections of $\bar{p}$ and p on protons, as well as of π^- and π^+ on protons, are considered. It is found that the requirement of the correct sign for each difference determines the value of δ^2, the integration cutoff over momentum transfer. The calculated $\bar{p}-p$ difference is too small to explain the observed values in the range 10 BeV/$c \leq P_L \leq 30$ BeV/c, and short-range contributions such as annihilation must still account for the difference. The $\pi^- - \pi^+$ difference with the correct sign (for $\delta^2=\mu^2$) is also too small, but for $\delta^2=4\mu^2$ and $P_L=4.5$ BeV/c it is more than twice the experimental value with the opposite sign. Since there is no annihilation effect for the $\pi^- - \pi^+$ difference, the process giving rise to the observed difference could even be twice as strong as previously suspected. Current notions concerning Regge poles indicate that one-ρ meson exchange should be most important.

INTRODUCTION

THE Pomeranchuk theorem concerning the equality of high-energy particle and antiparticle cross sections is well known.[1] However, all data to present, including laboratory momenta as high as 28 BeV/c for proton-proton collisions, indicate that cross section differences appear and are practically constant. Considering scattering on target protons, the differences of $\bar{p}-p$, $\pi^- - \pi^+$, and $K^- - K^+$ are all several millibarns.[2]

It seems reasonable that the strong, long-range interactions might account for these differences. The effects of one-pion exchange have been investigated by various authors[3,4] for several types of collisions. Similar considerations are here applied to determine the high-energy differences. Requiring the predicted values to have the correct sign seems to imply the choice of the maximum value of the square of the momentum transfer $\Delta^2_{\text{abs. max}} \equiv \delta^2$. Using $\delta^2=4\mu^2$ for the $\bar{p}-p$ case, the results indicate that the one-pion-exchange contribution is two orders of magnitude too small compared to the observed value. Thus it is concluded that short-range effects are needed to account for most of the difference in the 10- to 30-BeV energy range.

Carrying through similar calculations for the $\pi^- - \pi^+$ difference depends on a knowledge of the pion-pion cross section. It is found that the assumption of no pion-pion resonance explains approximately 10% of the observed value. When a pion-pion resonance is assumed, and the value of δ^2 determined by the $\bar{p}-p$ calculations is used, the result is a difference of more than twice that observed but of the opposite sign. If this result is taken seriously, it indicates that the mechanism operating to produce the observed difference must, in fact, be twice as large as previously expected. Finally, however, if a value of $\delta^2=\mu^2$ is used for consistency in the sign of the $\pi^- - \pi^+$ difference, the results are again only about 10% of the observed value.

II. SHORT DERIVATION OF $\Delta\sigma_{\bar{p}-p,p}$

In order to find the total cross section for nucleon-nucleon scattering, first consider the process indicated in Fig. 1. Nucleon p_1 is incident on nucleon p_2 at rest in the laboratory system. A pion of four-momentum Δ is exchanged, and a group of particles m emerges from one vertex while group n emerges from the other. It is, furthermore, assumed that m and n are always two well-defined groups. In the center-of-momentum system, one group is contained in a small forward angular cone, while the other is in a small backward cone. Identical particle symmetry is only provided between particles of the same group.

Using the Feynman rules we may write ($\hbar=c=1$)

$$S_{p_1,p_2}{}^{mn}=\frac{i}{(2\pi)^4}\int\frac{d^4\Delta}{(\Delta^2+\mu^2)}(2\pi)^3(2\omega_\Delta)(2\pi)^8 \times\delta(p_1+\Delta-p_m)\delta(p_2-\Delta+p_n) \times M_{p_1,\Delta}(m)M_{p_2,-\Delta}(n), \quad (1.1)$$

where $M_{p_1,\Delta}(m)$ is the invariant scattering amplitude for a pion and nucleon into the state m. $M_{p_2,-\Delta}(n)$ is similarly defined.

The Δ integral may be performed and $|S_{p_1p_2}{}^{mn}|^2$ can be calculated. It is then convenient to reintroduce the Δ integration with the minimum limit on Δ^2 determined

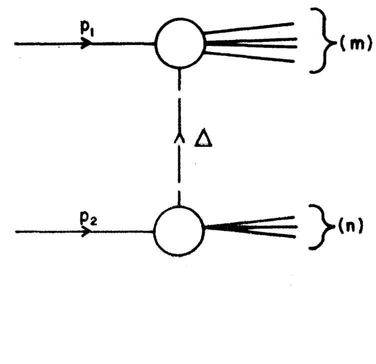

FIG. 1. One-pion-exchange contribution for inelastic scattering to two distinct final groups of particles m and n.

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* Supported in part by the U. S. Atomic Energy Commission.
[1] I. Ya. Pomeranchuk, Soviet Phys.—JETP 7, 499 (1958).
[2] K. Winter, International Conference on Theoretical Aspects of Very High-Energy Phenomena, 1961, CERN Report 61-22.
[3] For theoretical treatments see F. Salzman and G. Salzman, Phys. Rev. 125, 1703 (1962); and E. Ferrari and F. Selleri, CERN Report 2845/TH 243 (unpublished). The latter also contains a bibliography of applications of the theory.
[4] S. D. Drell, Revs. Mod. Phys. 33, 458 (1961).

by threshold single-pion production at each vertex. At present we shall only assume that the upper limit on Δ^2 be small, or not too far from the unphysical pole at $\Delta^2=-\mu^2$. It will be shown that conservation of total energy fixes the remaining two limits. Using final-state phase space $d\rho_f=d\rho_m d\rho_n$, and indicating the sum over final spins and average over initial spins by a bar over each scattering amplitude, the partial cross section may be written

$$\sigma_{p_1,p_2}{}^{mn}=\frac{(2\pi)^6}{v_{p_1,p_2}}\int\frac{d^4\Delta}{(\Delta^2+\mu^2)^2}d\rho_m d\rho_n(4\pi\omega_\Delta)^2 \times[(2\pi)^4\delta(p_1+\Delta-p_m)|\bar{M}_{p_1,\Delta}(m)|^2 \times(2\pi)^4\delta(p_2-\Delta-p_n)|\bar{M}_{p_2,-\Delta}(n)|^2], \quad (1.2)$$

where $v_{p_1p_2}$ is the relative velocity of the two nucleons. Since each vertex now closely resembles a meson-nucleon scattering process, it is convenient to define off-the-mass-shell cross sections

$$\sigma_{p_1,\Delta}(m)=\frac{(2\pi)^6}{v_{p_1,\Delta}}\int d\rho_m(2\pi)^4 \times\delta(p_1+\Delta-p_m)|\bar{M}_{p_1,\Delta}(m)|^2,$$
$$\sigma_{p_2,-\Delta}(n)=\frac{(2\pi)^6}{v_{p_2,-\Delta}}\int d\rho_n(2\pi)^4 \times\delta(p_2-\Delta-p_n)|\bar{M}_{p_2,-\Delta}(n)|^2. \quad (1.3)$$

Summing over all possible states m and n and charges of the exchanged pion, and using $\sigma_{p_1,\Delta}$ as a total pion-nucleon cross section, the total nucleon-nucleon cross section is

$$\sigma_{p_1,p_2}=\frac{4}{(2\pi)^4}\frac{1}{v_{p_1,p_2}}\sum_C\int\frac{d^4\Delta}{(\Delta^2+\mu^2)^2} \times[\omega_\Delta v_{p_1,\Delta}\sigma_{p_1,\Delta}][\omega_\Delta v_{p_1,-\Delta}\sigma_{p_1,-\Delta}], \quad (1.4)$$

where $\sum_C$ denotes the charge summation. Finally, multiplying through by $E_{p_1}E_{p_2}v_{12}$ the result is

$$\Gamma_{p_1,p_2}=\frac{4}{(2\pi)^4}\sum_C\int\frac{d^4\Delta}{(\Delta^2+\mu^2)^2}\Gamma_{p_1,\Delta}\Gamma_{p_2-\Delta}, \quad (1.5)$$

where Γ_{p_1,p_2} is the Lorentz invariant nucleon-nucleon combination $E_{p_1}E_{p_2}v_{p_1,p_2}\sigma_{p_1p_2}$, and the pion-nucleon vertices are similarly defined.

It is now to be assumed that $\Gamma_{p_1,\Delta}$ is a weak function of Δ^2 and can be well approximated by the experimentally measured on-the-mass-shell values; i.e., at $\Delta^2=-\mu^2$. Thus the dominant Δ^2 dependence in the integrand is attributed to the pole term. Further Δ^2 dependence occurs in the integration limits and will be discussed later. Furthermore, Salzman and Salzman[3] have shown for small Δ^2 the exchanged pion appears in the center of momentum of each vertex as if it were an incoming particle with positive energy. Thus, a π^+ incident on p_1 (in the rest frame of p_1, Δ, and p_m) also appears as a π^- incident on p_2 (in the rest frame of p_2, $-\Delta$, and p_n).

In Eq. (1.5) the charge labels have been suppressed. If p_1 and p_2 are both protons and the pion charge is denoted by a superscript on Γ, isotopic spin conservation allows the summation to be written as

$$\Gamma_{p_1,p_2}=\frac{4}{(2\pi)^4}\int\frac{d^4\Delta}{(\Delta^2+\mu^2)^2} \times[(5/4)\Gamma_{p_1,\Delta}{}^+\Gamma_{p_2,-\Delta}{}^-+(5/4)\Gamma_{p_1,\Delta}{}^-\Gamma_{p_2,-\Delta}{}^+ +\tfrac{1}{4}\Gamma_{p_1,\Delta}{}^+\Gamma_{p_2,-\Delta}{}^++\tfrac{1}{4}\Gamma_{p_1,\Delta}{}^-\Gamma_{p_2,-\Delta}{}^-]. \quad (1.6)$$

If p_1 is an antiproton, denoted by $\bar{p}_1$, while p_2 is a proton, charge conjugation invariance allows the vertices to be written in terms of protons and charged pions as follows:

$$\Gamma_{\bar{p}_1,p_2}=\frac{4}{(2\pi)^4}\int\frac{d^4\Delta}{(\Delta^2+\mu^2)^2}(\tfrac{1}{4}\Gamma_{p_1,\Delta}{}^+\Gamma_{p_2,-\Delta}{}^- +\tfrac{1}{4}\Gamma_{p_1,\Delta}{}^-\Gamma_{p_2,-\Delta}{}^++(5/4)\Gamma_{p_1,\Delta}{}^+\Gamma_{p_2,-\Delta}{}^+ +(5/4)\Gamma_{p_1,\Delta}{}^-\Gamma_{p_2,-\Delta}{}^-). \quad (1.7)$$

Finally the antiproton, proton difference is found to be

$$(\Delta\Gamma)_{p_1,p_2}=\frac{4}{(2\pi)^4}\int\frac{d^4\Delta}{(\Delta^2+\mu^2)^2} \times(\Gamma_{p_1,\Delta}{}^+-\Gamma_{p_1,\Delta}{}^-)(\Gamma_{p_2,-\Delta}{}^+-\Gamma_{p_2,-\Delta}{}^-). \quad (1.8)$$

III. KINEMATICS AND PHASE SPACE

To be more general, let p_1 and p_2 now represent particles of different mass. Conforming to the notation introduced by Salzman and Salzman,[3] we have

$$p_1^2=-w^2,$$

and

$$p_2^2=-w'^2.$$

Furthermore, it is convenient to introduce as variables the energy of the group m in the rest frame of the p_1 vertex and the energy of the group n in the rest frame of the p_2 vertex.

$$(p_1+\Delta)^2=-W^2,$$

and

$$(p_2-\Delta)^2=-W'^2.$$

The four integration variables $\Delta=(\Delta_1,\Delta_2,\Delta_3,\Delta_0)$ may now be changed to the new variables Δ^2, W^2, W'^2, and ϕ the azimuthal angle. If the Jacobian of the transformation is evaluated in the laboratory system, the result is

$$d^4\Delta=\Delta_{12}d\phi d\Delta_{12}d\Delta_3 d\Delta_0=(1/8mp_L)d\phi d\Delta^2 dW^2 dW'^2, \quad (2.1)$$

where $\Delta_{12}=(\Delta_1^2+\Delta_2^2)^{1/2}$ and p_L is the magnitude of $\mathbf{P}_1$ in the laboratory system ($|\mathbf{P}_2|=0$ in this system). The limits of integration for Δ^2 have already been mentioned. It is now necessary to discuss the limits of integration for W^2 and W'^2. It is possible to find relations between quantities in the several Lorentz frames mentioned. These are all listed by Salzman and Salzman and for the present purposes it is sufficient to merely quote the result:

$$\Delta^2=\tfrac{1}{2}\left[U^2-(w^2+w'^2+W^2+W'^2)+\frac{(w^2-w'^2)(W^2+W'^2)}{U^2}\right]-\frac{\cos\theta}{2U^2}[(U^2-w^2-w'^2)^2-4w^2w'^2]^{1/2}\times[(U^2-W^2-W'^2)^2-4W^2W'^2]^{1/2}, \quad (2.2)$$

where U is the total energy and θ the angle between $\mathbf{P}_1$ and $\mathbf{P}_m$ in the over-all barycentric system. It is then obvious that for given W^2 and W'^2, the minimum and maximum values of Δ^2 are given by $\theta=0$ and $\theta=\pi$, respectively. In particular, it is possible to write the equation as

$$\Delta^2=\Delta^2{}_{\min}(W^2,W'^2)+\frac{\sin^2(\theta/2)}{U^2}[(U^2-w^2-w'^2)^2-4w^2w'^2]^{1/2}[(U^2-W^2-W'^2)^2-4W^2W'^2]^{1/2}, \quad (2.3)$$

where $\Delta^2{}_{\min}(W^2,W'^2)$ is given by Eq. (2.2) with $\theta=0$. It is to be noted that the θ-dependent term in Eq. (2.3) is positive semidefinite in the physical range of θ, W^2, and W'^2. A closer inspection of $\Delta^2{}_{\min}(W^2,W'^2)$ reveals that it is an absolute minimum if $W^2=W'^2=w^2=w'^2$. It then has the value $\Delta^2{}_{\min}=0$. However, contributions from nonexcited vertices are fairly negligible.[4] For the particular cases of interest ($W^2=W'^2=m^2$), the minimum value of W^2 and of W'^2 is $(m+\mu)^2$ to just create a threshold pion at each vertex (m=nucleon mass; μ=pion mass). For nucleon-nucleon scattering the lower integration limit on Δ^2 is thus

$$\Delta^2{}_{\text{abs. min}}=\tfrac{1}{2}[U^2-2m^2-2(m+\mu)^2]-\tfrac{1}{2}[U^2-4m^2]^{1/2}[U^2-4(m+\mu)^2]^{1/2}. \quad (2.4)$$

The function $\Delta^2{}_{\min}(W^2,W'^2)$ has the further property that it is an increasing function of W^2 if W'^2 is held fixed. Furthermore, if $W'^2=W_{\min}'^2=(m+\mu)^2$, the maximum value of W^2 is obtained from the maximum value of $\Delta^2{}_{\min}$. Since the absolute maximum of Δ^2 is to be chosen by the requirement that it be not too far from the pole-value of $\Delta^2=-\mu^2$, the absolute maximum of W^2 is found by

$$\Delta^2{}_{\text{abs. max}}=\delta^2=\Delta^2{}_{\min}(W^2{}_{\max},W'^2{}_{\min}), \quad (2.5)$$

because the θ dependent term in Δ^2 is positive semidefinite. In fact, the role of W^2 and W'^2 are quite symmetrical for nucleon-nucleon collisions and the equation also defines $W_{\max}'^2$. Finally the values of W^2 and W'^2 which satisfy

$$\delta^2=\Delta^2{}_{\min}(W^2,W'^2) \quad (2.6)$$

form a boundary [with the lines $W^2=(m+\mu)^2$ and $W'^2=(m+\mu)^2$] in the W^2, W'^2 plane of integration. One should note that values of δ^2 under consideration are here implied to be, perhaps, as large as $9\mu^2$, in sharp contrast to the value $\Delta^2=O(U^2)$ allowed by conservation of energy near $\theta=\pi$. Thus, although values of $\Delta^2=O(U^2)$ are allowed, the fact that a large fraction of the cross section arises from a very small region of Δ^2 near the pole is really the basis of these calculations. Simple algebraic manipulation allows Eq. (2.6) to be written in a more convenient form for calculation. For the case of nucleon-nucleon collisions (with $W^2/m^2=x$, $W'^2/m^2=y$, and $\Delta^2{}_{\text{abs. max}}=\delta^2$) the result is

$$x^2+y^2+[(U/m)^2-2]xy-(U/m)^2(m^2-\delta^2)(x+y)+(U/m)^2[(m^2+\delta^2)^2-U^2\delta^2]=0. \quad (2.7)$$

As pointed out by Salzman and Salzman, the curve defined by Eq. (2.7) resembles one branch of an hyperbola, especially for large values of U^2. Figure 2 shows the results of calculating the defined boundaries at a fixed energy $U^2=20m^2$, which corresponds to $p_L=8.4$ BeV/c. It is especially to be noted that for fixed U^2, the enclosed area in the W^2, W'^2 plane increases with increasing δ^2. Although it is not shown graphically, it is also true that for fixed δ^2 the enclosed area increases with increasing U^2 in a similar fashion.

IV. $\bar{p}$-p CALCULATIONS

Using Eq. (2.1) and doing the Δ^2 integration, Eq. (1.8) may be written

$$\Delta\sigma_{\bar{p}-p,p}(p_L)=\frac{1}{2(2\pi)^3(mp_L)^2}\left[\frac{1}{\Delta^2{}_{\text{abs. min}}+\mu^2}-\frac{1}{\delta^2+\mu^2}\right]\times\int_A dW^2dW'^2[mp_L(\sigma_+-\sigma_-)]_{W^2}\times[mp_L(\sigma_+-\sigma_-)]_{W'^2}, \quad (3.1)$$

where $\sigma_\pm$ are the total $\pi^\pm$-nucleon cross sections. The combination $(\Gamma_{p_1,\Delta}{}^+-\Gamma_{p_1,\Delta}{}^-)$ is evaluated in the laboratory system for pion-nucleon scattering, and Δ^2 is set equal to $-\mu^2$. A change of the energy variable is then performed to W^2 which is, in fact, the total energy in the pion-proton barycentric system. Using the experimental data,[5–8] Fig. 3 shows the results of the change

[5] S. J. Lindenbaum and L. C. L. Yuan, Phys. Rev. **100**, 306 (1955).

[6] R. Cool, O. Piccioni, and D. Clark, Phys. Rev. **103**, 1082 (1956).

[7] M. Longo, J. A. Helland, W. N. Hess, B. J. Moyer, and V. Perez-Mendez, Phys. Rev. Letters **3**, 568 (1959).

[8] G. von Dardel, R. Mermod, P. A. Piroue, M. Virargent, G. Weber, and K. Winter, Phys. Rev. Letters **7**, 127 (1961); **8**, 173 (1962).

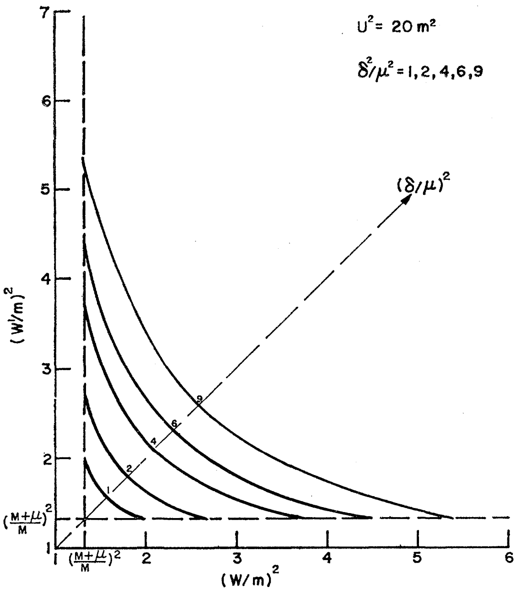

FIG. 2. Phase-space boundaries in the W^2, W'^2 plane for several values of δ^2. $U^2=20m^2$ corresponds to $p_L=8.4$ BeV/c.

of variable. Several features are to be noticed; in particular: (1) $\sigma_+-\sigma_-$ near threshold is practially zero, (2) the 3-3 resonance gives a large positive contribution, but it is somewhat diminished by the factor p_L near threshold, (3) the second resonance is just as important as the 3-3 resonance because p_L is further from threshold; furthermore, it has the opposite sign.

Due to the strong U^2 dependence appearing in Eq. (3.1) as

$$\frac{1}{(mp_L)^2}=\frac{4}{U^2(U^2-4m^2)}\sim\frac{4}{U^4} \quad \text{for large } U^2,$$

it is reasonable to do the calculations at the low values of U^2; $\Delta\sigma_{\bar{p}-p}$ could only be predicted constant if the integral somehow showed a U^2 dependence. Once more it is to be understood that δ^2 is presumably in the range $\mu^2\leq\delta^2\lesssim 9\mu^2$ if the model is to be plausible. In fact, previous calculations in other applications of the model have extended the integration for $\delta^2>50\mu^2$.[9] In most cases the weight function over which the integral is performed was positive definite, i.e., the product of two physical cross sections. However in the present calculation, Fig. 3 has indicated the large negative contributions possible in the integrals.

Due to the possibility of large negative contributions to the integral, a self-consistent determination of δ^2 is suggested. The calculations of $\Delta\sigma_{\bar{p}-p,p}$ must yield not only the correct magnitude but also the correct sign. A study of Fig. 4 should indicate the situation more clearly. This is a reproduction of Fig. 2 with additional information about the weight functions. Along each axis

TABLE I. Proton-antiproton cross-section differences for several values of δ^2.

$(\delta/\mu)^2$	$[\Delta\sigma_{\bar{p}-,p}({}_pU^2=20m^2)]$ (mb)
1	0.0001
2	0.102
4	0.148
6	−0.247
9	−0.617

the three regions of $p_L(\sigma_+-\sigma_-)$ to be noted are: (1) the threshold strips, (2) the positive 3-3 resonance strips, and (3) the predominantly negative strips of the higher resonance. On the W^2, W'^2 plane are indicated the regions of positive and negative contributions from the product of the two factors. It is to be noted that the two negative wing areas contribute appreciably for $\delta^2>4\mu^2$. Thus, the graph should indicate that $\delta^2=\mu^2$ would give the correct sign but would not give the maximum value, while $\delta^2=9\mu^2$ will certainly contain too much negative contribution.

The results of the actual calculations for the several values of δ^2 are shown in Table I. It is thus found that $\delta^2\simeq 4\mu^2$ allows a maximum difference in the cross sections with the correct sign. Further increase in δ^2 quickly produces a change in the sign and an increase in magnitude. The value of 0.15 mb at $U^2=20m^2$ is, however, quite inadequate to explain the large values observed (order of 10 mb). Furthermore, once δ^2 is chosen equal to $4\mu^2$, the calculations at $U^2>20m^2$ will quickly turn negative. This is simply a consequence of the fact noted above that the boundary curve moves outward in the W^2, W'^2 plane for an increase in U^2 just as in δ^2. In addition, the $1/U^4$ dependence outside the integral serves to rapidly diminish the magnitude of $\Delta\sigma_{\bar{p}-p,p}$. Calculations with δ^2 fixed at $4\mu^2$ and $U^2=40m^2$ ($p_L\simeq 17.8$ BeV/c) indicate that $\Delta\sigma_{\bar{p}-p,p}$ is approximately equal to -0.05 mb.

As a final calculation for the $\bar{p}-p$ difference, δ^2 can be assumed to be a function of U^2 such that the area in the W^2, W'^2 plane remains constant and equal to that which gives the positive maximum found for $U^2=20m^2$

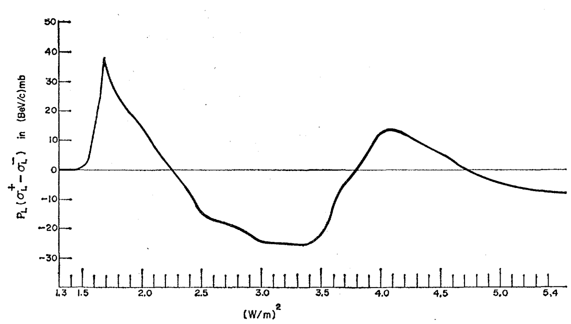

FIG. 3. The difference of the invariant $\pi^\pm$-nucleon scattering combinations as a function of the square of the total barycentric energy.

[9] For example, see I. M. Dremin and D. S. Chernevskii, Soviet Phys.—JETP 11, 167 (1960); 12, 94 (1961).

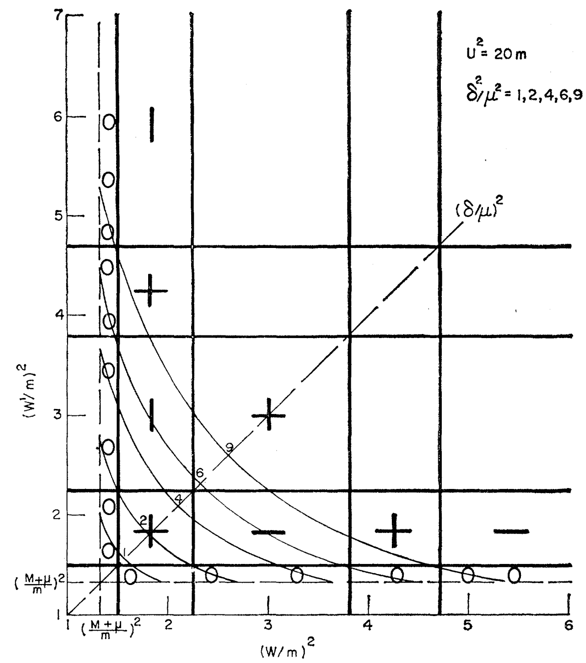

FIG. 4. Figure 2 is reproduced indicating the regions of positive and negative contributions from the product of the two $\pi-N$ invariant weight functions.

and $\delta^2=4\mu^2$. This function is most easily approximated by using Eq. (2.6) fitted for $W^2=W'^2=2.1m^2$; this point lies on the boundary for $U^2=20m^2$ and $\delta^2=4\mu^2$. The result of this substitution is

$$\delta^2(U^2)=-3.1m^2+\tfrac{1}{2}[U^2-(U^2-4m^2)^{1/2}\times(U^2-8.4m^2)^{1/2}], \quad (3.2)$$

for $U^2\geq 8.4m^2$. As $U^2\to\infty$, this equation implies $\delta^2(U^2)\to 0$. Table II indicates several values of $\delta^2(U^2)$ computed from Eq. (3.2).

Using the value of the integral computed at $U^2=20m^2$ and $\delta^2=4\mu^2$, the cross-section difference can be scaled as a function of U^2 to yield

$$\Delta\sigma_{\bar{p}-p,p}=(20.7)\frac{1}{(U/m)^2[(U/2m)^2-1]}\times\left(\frac{\delta^2(U^2)}{\delta^2(U^2)+\mu^2}\right)\text{ mb}, \quad (3.3)$$

where $\Delta^2_{\text{abs. min}}$ has been set equal to zero. The results of calculations with Eq. (3.3) are also shown in Table II (rounded to two decimal places). It is obvious that the U^{-4} dependence is overwhelming; using δ^2 as a function of U^2 to maintain the maximum positive phase space integral serves only to keep the correct sign, but the magnitude still decreases rapidly.

Briefly, there are two viewpoints to interpret these calculations. If it is assumed that the cutoff value $\delta^2=4\mu^2$ at $U^2=20m^2$ is determined by requiring the correct sign in $\Delta\sigma_{\bar{p}-p,p}$, then the resulting magnitude is quite inadequate to explain the observed values. If, on the other hand, values of $\Delta^2\simeq 9\mu^2$ or larger but still not extremely far from the pole at $-\mu^2$ are allowed, the calculations indicate that the peripheral contributions are nearly one millibarn of the opposite sign. Thus, the actual mechanism accounting for the measured values of $\Delta\sigma_{\bar{p}-p,p}$ would have to be slightly stronger than previously supposed. If a mechanism were found to account for a difference of 10 mb, it is not difficult to imagine it could equally well account for 11 mb, or a 10% increase. Such a process is usually thought to be the strong, short-range annihilation effect. If annihilation[10] is assumed to explain only the major part of the difference in the cross sections at 8.4 BeV/c, then the remaining, undisclosed mechanism would have to be stronger than previously suspected in order to cancel the negative peripheral contributions. In the case of the $\pi^--\pi^+$ difference, which will now be treated, the annihilation channel does not exist, and the necessity of a stronger but yet undisclosed channel is perhaps more clear.

V. $\pi^--\pi^+$ CALCULATIONS

A derivation following that schematically presented in the $\bar{p}-p$ case may now be performed for pions incident on protons. In Fig. 1, p_1 is to be considered a pion with incident laboratory momentum equal to p_L. The result may again be written as in Eq. (1.5) where $\Gamma_{p_1p_2}=\omega_{p_1}E_{p_2}v_{p_1,p_2}\sigma_{p_1p_2}$, and $\Gamma_{p_1,\Delta}$ is a pion-pion interaction. If the two pion charges at the p_1 vertex are indicated by a double superscript on $\Gamma_{p_1,\Delta}$, the charge summation for an incident π^+ leads to

$$\Gamma_{p_1^+p_2}=\frac{4}{(2\pi)^4}\int\frac{d^4\Delta}{(\Delta^2+\mu^2)^2}\{(\Gamma_{p_1,\Delta}{}^{+-}+\tfrac{1}{2}\Gamma_{p_1,\Delta}{}^{+0})\Gamma_{p_2,-\Delta}{}^{+}+(\Gamma_{p_1,\Delta}{}^{++}+\tfrac{1}{2}\Gamma_{p_1,\Delta}{}^{+0})\Gamma_{p_2,-\Delta}{}^{-}\}. \quad (4.1)$$

After the summation for an incident π^- is performed,

TABLE II. Table of "optimum" δ^2 and resulting proton-antiproton cross-section differences for various total energies.

$(U/m)^2$	$[\delta(U^2)/\mu]^2$	$(\Delta\sigma_{\bar{p}-p,p})$ (mb)
20	4.0	0.15
30	2.8	0.06
40	1.8	0.04
50	1.3	0.02
100	0.6	<0.01
200	0.3	
500	0.1	

[10] An estimate of the annihilation cross section at 2 BeV/c is as high as 25 mb.; see R. Armenteros, C. A. Coombes, B. Cork, G. R. Lambertson, and W. A. Wenzel, Phys. Rev. 119, 2068 (1960).

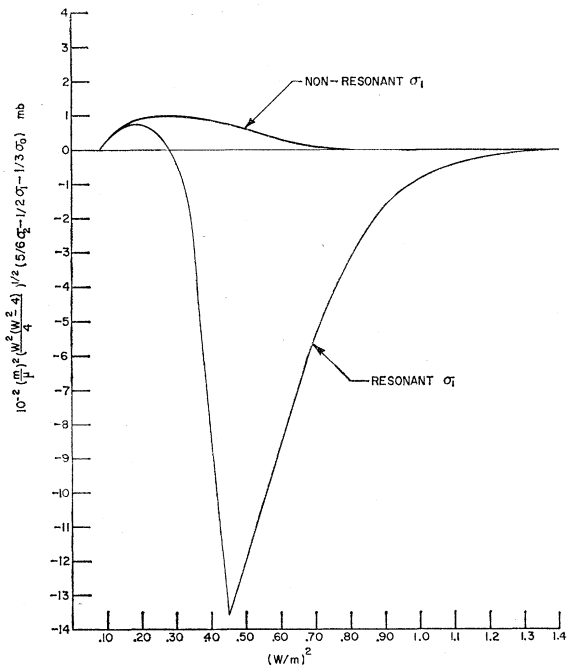

FIG. 5. The invariant $\pi-\pi$ weight function for nonresonant and resonant σ_1 as a function of the square of the total barycentric energy.

the $\pi^- - \pi^+$ difference is found to be

$$(\Delta\Gamma)_{p1^- - p1^+, p2} = \frac{4}{(2\pi)^4} \int \frac{d^4\Delta}{(\Delta^2+\mu^2)^2} (\tfrac{5}{6}\Gamma_{p1,\Delta}{}^{(2)} - \tfrac{1}{2}\Gamma_{p1,\Delta}{}^{(1)} - \tfrac{1}{3}\Gamma_{p1,\Delta}{}^{(0)})(\Gamma_{p2,-\Delta^+} - \Gamma_{p2,-\Delta^-}), \quad (4.2)$$

where the superscript (2), (1), and (0) on $\Gamma_{p_1,\Delta}$ now indicates the isotopic spin of the two-pion system.

Kinematic and phase-space calculations are quite similar to those for the $\bar{p}-p$ case; however, they are not as symmetrical. In particular the two variables W^2 and W'^2 are again to be introduced. Equation (2.1) remains valid, and Eq. (2.2) is to be used with $w^2=\mu^2$ and $w'^2=m^2$. Exactly the same arguments presented in Sec. II lead to

$$\Delta^2_{\text{abs. min}}(U^2) = \Delta^2[U^2, \theta=0, W^2=(2\mu)^2, W'^2=(m+\mu)^2], \quad (4.3)$$

and the boundaries in the W^2, W'^2 plane are

$$\Delta^2_{\text{abs. max}} \equiv \delta^2 = \Delta^2_{\min}(W^2, W'^2), \quad W^2=(2\mu)^2, \quad W'^2=(m+\mu)^2. \quad (4.4)$$

If we denote the total pion-pion cross section in the state of isotopic spin T by σ_T, Eq. (4.2) becomes

$$\Delta\sigma_{\pi^- - \pi^+, p} = \frac{1}{2(2\pi)^3 (mp_L)^2} \left[\frac{1}{\Delta^2_{\text{abs. min}}+\mu^2} - \frac{1}{\delta^2+\mu^2} \right] \times \int dW^2 dW'^2 \left[\left(\frac{W^2(W^2-4)}{4} \right)^{1/2} (\tfrac{5}{6}\sigma_2 - \tfrac{1}{2}\sigma_1 - \tfrac{1}{3}\sigma_0) \right]_{W^2} \times [mp_L(\sigma_+ - \sigma_-)]_{W'^2}, \quad (4.5)$$

where the Lorentz invariant $\pi-\pi$ combination is most easily considered in its own center of mass, and the $\pi-p$ combination has already been discussed.

In order to proceed further, the $\pi-\pi$ cross sections must be known. Referring to the work by Anderson *et al.*,[11] it will be assumed that $\sigma_0 \simeq \sigma_2 = 70$ mb at threshold. Also it is assumed that all σ_T asymptotically approach 35 mb at high energy; in particular σ_2 and σ_0 monotonically decrease to 35 mb. The present calculations are not sensitive to possible high energy differences of a few mb compared to 35 mb. It is then possible to use the results of Anderson *et al.* to infer the resonance form of σ_1. For comparison a nonresonant form for σ_1 was also used. The pion-pion weight function appearing in Eq. (4.5) is shown for these two cases in Fig. 5. Note that the nonresonant curve is everywhere positive, but the resonant curve is almost everywhere negative except near threshold.

Assuming that $\delta^2=4\mu^2$, as determined for the $\bar{p}-p$ case, is characteristic of the one-pion exchange in general, Fig. 6 indicates the region on the W^2, W'^2 plane over which the integral is to be performed for $U^2=10m^2$ ($p_L=4.2$ BeV/c). Also shown are the regions of positive and negative contributions from the product of the pion-nucleon and resonant pion-pion factors. Even be-

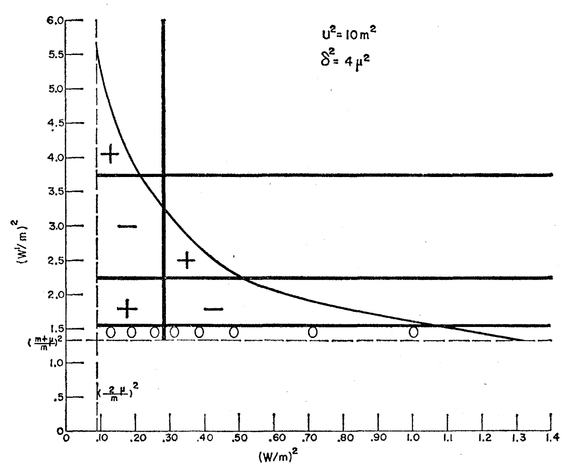

FIG. 6. Phase-space boundaries in the W^2, W'^2 plane for $\delta^2=4\mu^2$, $U^2=10m^2$ ($p_L=4.2$ BeV/c). Regions of positive and negative contributions from the product of the $\pi-N$ and resonant $\pi-\pi$ invariant weight functions are indicated.

[11] J. A. Anderson, Vo X. Bang, P. G. Burke, D. D. Carmony, and N. Schmitz, Revs. Mod. Phys. 33, 431 (1961).

TABLE III. $\pi^- - \pi^+$ cross-section differences in mb for two values of δ^2 and for two models described in the text.

	$\Delta\sigma_{\pi^- - \pi^+, p}$	
Case	$\delta^2 = 4\mu^2$	Threshold ($\delta^2 \sim \mu^2$)
Resonant	-6.5	$+0.2$
Nonresonant	$+0.03$	$+0.3$

fore the calculations are performed, Fig. 6 strongly suggests that $\delta^2 = 4\mu^2$ allows too much negative contribution. Thus, the maximum positive contribution due only to the $\pi - \pi$ threshold and $\pi - N$ (3,3) resonance is also considered. This positive contribution corresponds roughly to an integration cutoff of $\delta^2 = \mu^2$. The results of these calculations are shown in Table III.

The observed value of $\Delta\sigma_{\pi^- - \pi^+, p} = 2.6$ mb at $p_L = 4.5$ BeV/c cannot be fitted by these calculations. As in the $\bar{p} - p$ problem, the maximum positive value is not adequate to explain the difference in the cross sections. However, if δ^2 is increased to only $4\mu^2$, which is still quite near the pole, the resonant $\pi - \pi$ cross section implies a value more than twice the observed magnitude but of the incorrect sign. Since there is no short-range annihilation process in this case, the situation is now even more puzzling. If the one-pion-exchange contributions are to be seriously considered for $\delta^2 \sim 4\mu^2$, the mechanism sought to explain the large observed value of $\Delta\sigma_{\pi^- - \pi^+, p}$ must, in fact, be two or three times stronger than previously expected in order to cancel the peripheral contributions.

VI. CONCLUSIONS AND ACKNOWLEDGMENTS

The above calculations have indicated that by considering the high-energy difference of particle and antiparticle cross sections in the one-pion-exchange model, a self-consistent determination of the sign implies too small a magnitude for the difference. Although this result seems rather discouraging, it is, in fact, in agreement with the current notions concerning the importance of Regge poles in high-energy cross sections. Udgaonkar[12] has listed the Regge pole trajectories entering in various combinations of cross sections. In particular, the pion trajectory does not contribute to any of the particle, antiparticle differences. Since the $\pi^- - \pi^+$ difference involves only the ρ-meson trajectory, it would be interesting to further investigate the peripheral model for one-ρ-meson exchange. Should this channel be effective in fitting the observed values, it would also indicate the consistency of our choice of δ^2 in the above calculations.

The author wishes to thank Professor Ernest M. Henley for suggesting the problem considered and for several very helpful discussions while the work progressed.

[12] B. M. Udgaonkar, Phys. Rev. Letters 8, 142 (1962).